Stochastic sediment–vegetation dynamics in an Alpine braided river

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Abstract We propose a stochastic modelling framework to simulate the spatially lumped sediment and vegetation dynamics in a flood plain of a braided gravel bed river as an alternative to detailed morphodynamic modelling. The idea is that floods intermittently erode the river bed and expose sediment on gravel bars, while riparian vegetation continuously recolonises the exposed area. The exposed sediment and water stochastic dynamics can be analytically solved in terms of their probability density function (pdf), and later used to force a vegetation model, together forming a so-called coupled "Master–Slave" stochastic dynamical system in continuous time. We apply and validate the master model by using a historical record of aerial photographs of the Maggia River (Tessin, Switzerland) and daily streamflow measurements. This approach is useful to statistically quantify the amount of sediment that is reworked by flood events, as well as the effects of changes in the flood disturbance regime on flood-plain dynamics.

Key words flood plain processes; river–vegetation interactions; sediment dynamics; stochastic models

INTRODUCTION

The interactions between fluvial and ecological processes in the alluvial zone of Alpine braided rivers are key at both basin and reach scales. At the basin scale, for instance, the river and sediment dynamics reflect the mechanisms of erosion, transport and deposition that determine the statistical equilibrium conditions of the basin from both geomorphological (e.g. mean river slope, grain size distribution, soil stratification, etc.) and hydrological (e.g. river and infiltration dynamics, groundwater recharge and flow, etc.) viewpoints. At the local reach scale, sediment dynamics frequently modify the alluvial zone by erosion and deposition processes as a function of inundation frequency and flow competence. This results in the exposure of new sediment surfaces and nutrient cycling, which create favourable conditions for the establishment and development of different types of riparian vegetation (e.g. herbaceous, shrubs, trees) and support ecological habitat diversity (Resh *et al.*, 1988; Tockner *et al.*, 2000). Moreover, flood events also play the role of "natural cleaners" for the ecotone, preventing vegetation from invading the whole riverine corridor. Hence, the importance of studying such dynamics under changing conditions, due to either anthropogenic or climatic causes, or their joint interaction.

Many Alpine rivers have been subject to water withdrawals for different water-use purposes (e.g. energy, municipal, agricultural, etc.), which typically remove the seasonal component from the natural streamflow regime. Undoubtedly, the new regime affects the dynamics of transport processes (sediment, nutrients, debris, etc.), river morphology, groundwater recharge, and fluvial ecotone biodiversity in the riparian zone (Leyer, 2005). This is also the case in the Maggia Valley (Canton Tessin, southeastern Switzerland), where the natural streamflow regime has been reduced to a practically constant baseflow release (EFR), punctuated by occasional floods due to either extreme rainfall or sporadic controlled releases from dams, or both. The composition and extent of riparian vegetation in the valley flood plain changes in response to streamflow variability, as can be seen in Fig. 1 (Molnar *et al.*, 2008).

The processes occurring in the riparian zone are intrinsically complex and still poorly understood because of the numerous interactions and feedbacks between erosion, sedimentation, vegetation colonization, uprooting, etc. The study of the sediment dynamics in such environments must consider the active role of riparian vegetation in recolonising the gravel bars and islands that have been reworked by both erosion and sedimentation processes during flood events. Because of the coarse gravel material, only pioneer plants such as willow shrubs and herbaceous species, usually flourish in this zone, establishing and developing rapidly between flood events. In those

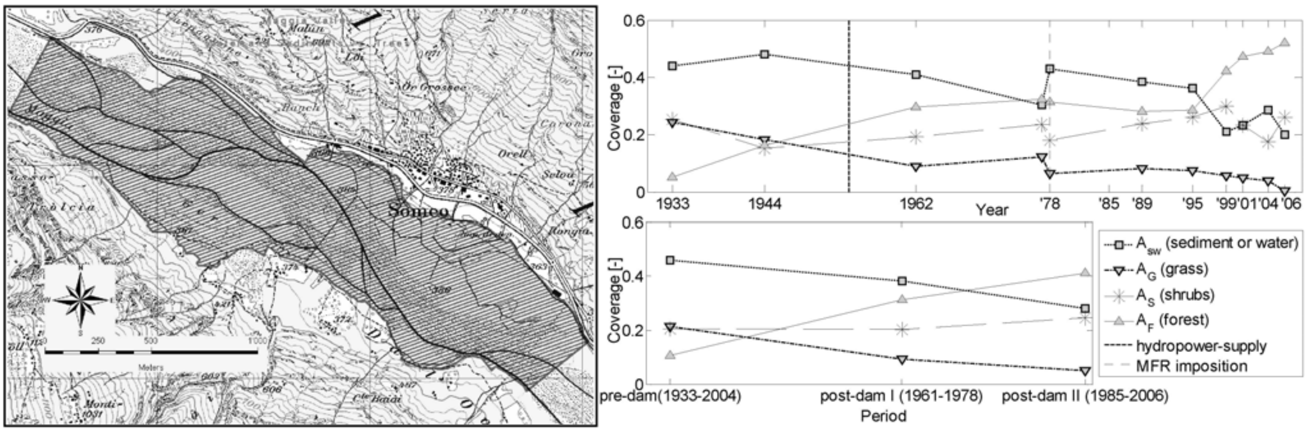

Fig. 1 Maggia valley, Canton Tessin, Switzerland. The left panel shows the braided reach of the Maggia River in the vicinity of Someo; the hatched area is chosen as the spatial domain for the lumped model. The right panel shows the trend of the different classes of morphological features in the flood plain. The vertical black dashed line indicates the approximate start year of the hydropower system operation in the valley.

environments where the groundwater system may not strictly depend on river flow (e.g. Alpine valleys where groundwater recharge often originates from side valleys and slopes), vegetation growth is almost unaffected by water withdrawal. Gradually, the exposed sediment surfaces may be either partially or entirely colonized into more-or-less stable island patterns, which form the riparian corridor (Decamps, 1993). Some indication about the effects of hydrological changes (of either climatic or anthropogenic origin) can be obtained by studying the way freshly exposed areas are recolonised (e.g. see Decamps, 1993). Physically-based modelling of this process is complicated because the available models generally do not contain a full process description including all relevant interactions and feedbacks. Often excessive parameterization is used, and because uncertainties also arise in parameter calibration, the consequent predictive capability of such models is limited to short-term predictions. We argue that a minimalist, physically-based model may be preferable when the dynamics of environmental systems are to be investigated over the medium to long term; see recent efforts by e.g. Lytle & Merritt (2004) and Camporeale & Ridolfi (2006).

In this work we propose such a minimalist modelling framework to describe the spatially-lumped water and sediment dynamics of Alpine rivers. In particular, we aim at a coupled model consisting of a stochastic component (*Master model*), which describes the dynamics of the exposed sediment and water surface lumped at the river-reach scale. The dynamics of the complementary area occupied by vegetation is split into three main classes that evolve according to a Master-dependent model (*Slave model*). Although we focus on the statistical properties of the Master model in this paper, we also discuss the philosophy underlying the Slave model and its purpose. As far as the Master model is concerned, we end up with a model that is parsimonious in the number of parameters which can be reliably estimated from aerial photography. From a quantitative viewpoint, this approach allows for a fully theoretical formulation of the process with an analytical solution. The main variable of interest is the percentage of area that will be covered by riparian vegetation under given hydrological and climatic conditions. We apply the Master model to the River Maggia, where flow regulation for hydropower purposes has heavily affected the streamflow regime, and where we have a number of historical aerial georeferenced images of the flood plain forest. We first test the model assumptions and simplifications, and then we estimate the probability of having a given exposed area as a function of the streamflow regime.

This paper is organized as follows: the next section deals with the description of the general modelling framework and the development of the main mathematical aspects of the Master model. In the Results and Discussion section we apply the Master model to the Maggia River and show its performance for a number of example simulations. We point out both advantages and disadvantages of our approach, and outline potential directions for future work.

MODELLING FRAMEWORK

We aim to explain sediment and vegetation dynamics in Alpine rivers as a consequence of stochastic disturbances by floods which rework the channel and flood plain, and the deterministic recolonisation of the exposed sediment by riparian vegetation. The evolution of each morphological and vegetation class can generally be obtained from georeferenced aerial photographs (see Fig. 1 and Molnar *et al.*, 2008, for an example of the Maggia River). A lumped representation of the flood-plain dynamics helps us to simplify the spatial and temporal complexity of the real process, but it nonetheless requires specific hypotheses in order to be applicable. These are:

1. the process is active only in the period T_w (i.e. April–November), whereas it is inactive during the winter low-flow period T_c (i.e. December–March);
2. flood disturbance occurs as a stochastic process, as it reasonably appears to be the case in the Maggia post-dam period, while colonization occurs deterministically;
3. the alluvial sediment material has poor cohesive capacity, and it is potentially mobilized by flows that locally exceed the critical conditions for bedload transport;
4. the area of exposed sediment and water simply increases as a consequence of an instantaneous stochastic flood event q exceeding a given threshold q^* and inundating an area $A(q)$. The corresponding erosion of the already vegetated channel occurs in successive stages (Fig. 2), i.e. grass is destroyed first, then shrubs and then forest, provided that the flood is large enough;
5. the colonization dynamics occur in successive stages (Fig. 2), i.e. first grass will colonize the available exposed sediment area A_{sw}, which then gradually turns into shrubs and then into forest.

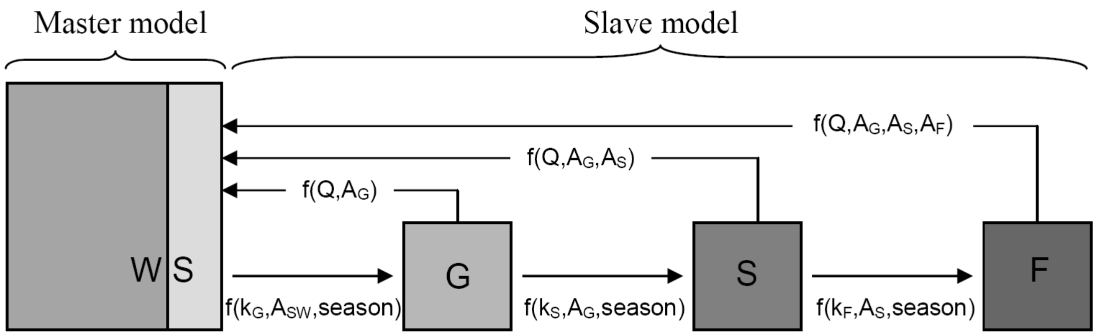

Fig. 2 Scheme of the Master–Slave modelling framework. The Master model describes the dynamics of sediment and water (SW), and is used to drive the Slave model which models the dynamics of the three classes of vegetation (G = grass or low stage vegetation; S = shrubs or middle stage vegetation; F = forest or high stage vegetation).

We propose a model whose origins are in dynamical system theory (e.g. see Sprott, 2003), i.e. a "Master–Slave" model, where the Master component is an idealized stochastic process in continuous time (Cox & Miller, 1965). The modelling scheme is shown in Fig. 2. We assume that our system can be studied at the medium to long term assuming statistical equilibrium conditions. This way, the fluctuations imposed by the stochastic modelling component reflect the natural system variability and are the result of inherent system complexity. In the following, we provide the model equations, referring the reader to Perona *et al*. (2008) for details about the derivation and related mathematical properties of the model. We denote with A_G the surface of the flood plain occupied by low-stage vegetation (e.g. grass and small plants), A_S the surface occupied by middle-stage vegetation (e.g. shrubs and young trees), and A_F the surface occupied by adult vegetation (e.g. forest with mature trees). The total vegetated area is $A_v = A_G + A_S + A_F$, which together with that of sediment and water A_{sw} complements the floodable area in the domain A_f. By defining $A_{i,j} = A_i + A_j$ the model assumes the following mathematical form, equations (1)–(4):

$$\frac{dA_{sw}}{dt} = \Theta(\Delta A_{sw}) \cdot \Delta A_{sw} - C_G \tag{1}$$

$$\frac{dA_G}{dt} = C_G - C_S - \Theta(\Delta A_{sw}) \cdot \left[\Theta(A_G - \Delta A_{sw}) \cdot \Delta A_{sw} + \Theta(\Delta A_{sw} - A_G) \cdot (A_G)\right] \tag{2}$$

$$\frac{dA_S}{dt} = C_S - C_F - \Theta(\Delta A_{sw}) \cdot \Theta(\Delta A_{sw} - A_G) \cdot \left[\Theta(A_{G,S} - \Delta A_{sw}) \cdot (\Delta A_{sw} - A_G) + \Theta(\Delta A_{sw} - A_{G,S}) \cdot (A_S - 1)\right] \tag{3}$$

$$\frac{dA_F}{dt} = C_F - \Theta(\Delta A_{sw}) \cdot \Theta(\Delta A_{sw} - A_{G,S}) \cdot \left[\Theta(A_{G,S,F} - \Delta A_{sw}) \cdot (\Delta A_{sw} - A_{G,S}) + \Theta(\Delta A_{sw} - A_{G,S,F}) \cdot (A_F - 1)\right] \tag{4}$$

where $C_G = k_G \cdot A_{sw}$, $C_S = k_S \cdot A_G$, and $C_F = k_F \cdot A_S$ are the colonization rates of the low-stage, of the middle-stage and of the adult vegetation, respectively. $\Delta A_{sw} = A(q) - A_{sw}$ defines the flooding event potential in exposing new sediments and $\Theta(\cdot)$ is the Heaviside step function that conditionally activates the different terms of the equation. Equations (1)–(4) are coupled to each other. Equation (1) is independent from equations (2)–(4) and defines the Master model. Equation (1) expressively accounts for the low-stage (grass) vegetation colonization dynamics, while equations (2)–(4) describe the evolution of the single classes of vegetation as a function of the current disturbances and the actual availability of exposed sediment and water. Because the latter information comes from the Master model, then equations (2)–(4) are driven by it and therefore define the "Slave" model. Altogether, the system of equations (1)–(4) defines a stochastic Master–Slave model in continuous time, whose dynamics, as far as the Master model is concerned, can be cast in probabilistic terms and then solved analytically by means of the theory of stochastic processes. This is discussed in the next subsection.

The Master model for the sediment and water dynamics

At a given time, the river reach shown in Fig. 1(a) has a total floodable surface A_f, a portion of which is made up of sediment and water A_{sw}, and the remainder, A_v, is occupied by vegetation, i.e. $A_f = A_{sw} + A_v$. We study the evolution of A_{sw} resulting from the action of sporadic flood disturbances q and the subsequent tendency of vegetation to recolonise the exposed area. Erosion and deposition are assumed to occur during floods above a given threshold q^*. The distribution of the event magnitude $q \geq q^*$ is assumed to be exponential, i.e. $b(q) = \lambda e^{-\lambda(q-q^*)}$, with mean $\mu_q = (1+\lambda q^*)/\lambda$ and variance $\sigma_q^2 = (2+\lambda q^*(2+\lambda q^*))/\lambda^2$, and λ being the only fitting parameter. Because the inter-arrival time among events τ is also exponentially distributed, a sequence of such disturbances defines a Poisson process of rate ρ (e.g. see Cox & Miller, 1965), which is the inverse of the average inter-arrival time $\tau = 1/\rho$.

The flood disturbance events affect an area A, which can readily be obtained from the hydraulic rating curve $A(q)$ for the flood plain (Fig. 3). This relationship may be obtained from a detailed 2D-hydraulic model (Ruf *et al.*, 2008), and it allows us to set the modelling approach on a sound physical basis. We assume the rating curve to be a power law $A(q) = k_1 q^{k_2}$ with k_1 and k_2 as fitting parameters, upper-limited at A_f when the flow q_f floods the entire domain. Although the rating curve is expected to change locally when flood events modify the river morphology, we assume it does not change in the upper part, i.e. for streamflow values $q \geq q^*$. By means of a derived distribution approach (see Perona *et al.*, 2008), we obtain the probability density function of the areas reworked by flood events $g(A)$. This is made of a continuous part and of an atom of finite probability for $A = A_f$:

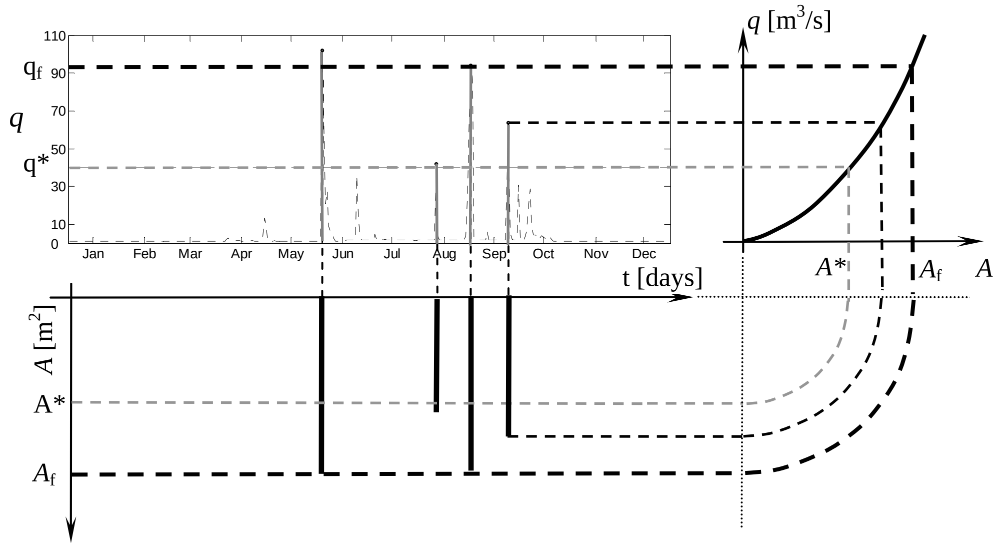

Fig. 3 Effect of disturbances on the erosion and deposition process. By means of the rating curve, flood events q above the threshold q^* may expose an area A conditionally to the current exposed area $A_{\rm sw}$.

$$g(A) = g^c(A) + g^{at}(A_f) = a_0 A^{a_1} e^{-(a_2 A^{1/k_2})} + e^{a_2 (A^{*1/k_2} - A_f^{1/k_2})} \tag{5}$$

In equation (5) the constants a_0, a_1, and a_2 are the result of algebraic simplification; their expressions can readily be derived or taken from Perona *et al.* (2008). This distribution can be integrated in order to obtain the cumulative probability distribution $G(A)$ which must satisfy the integral condition $\int_{A^*}^{A_f} g(A)dA = 1$

In order to proceed with the probabilistic formulation of the process we first rewrite equation (1):

$$\frac{dA_{sw}}{dt} = \sum_i \Theta(A(t_i) - A_{sw})(A(t_i) - A_{sw})\delta(t_i) - k_G A_{sw} \tag{6}$$

which shows that the first term on the right hand side is the stochastic disturbance A occurring at the generic time $t_{\rm i}$ with a probability of occurrence in the time interval dt equal to ρdt, and applied as an instantaneous event through the Dirac delta function $\delta(\cdot)$. This Poisson process is marked (e.g. Rodriguez-Iturbe *et al.*, 1999) in that the event magnitude is also random and follows the exponential distribution $b(q)$ (Fig. 4(a),(b)). Memory effects due to consecutive events are therefore not directly described in the erosion process. However, these are partially accounted for by using a time (e.g. daily) average instead of the actual flood peak when defining the magnitude of the stochastic disturbances. Notice, that equation (6), which defines the evolution equation of the process, shows disturbances being effective conditionally to the value of the current exposed area $A_{\rm sw}$ (Fig. 4(c)). The stationary properties of the stochastic process (6) are described in Perona *et al.* (2008), together with the mathematical details that lead to the Kolmogorov master equation describing the continuous part of the probability density function $p^{\rm c}(A_{\rm sw})$ of the process:

$$\frac{d(k_G A_{sw} p^c(A_{sw}))}{dA_{sw}} + \rho p^c(A_{sw}) - \rho p^c(A_{sw}) G^c(A_{sw}) - \rho g^c(A_{sw}) \int_0^{A_{sw}} p^c(a)da = 0\,, \tag{7}$$

for $A_{\rm sw} \in [0, A_{\rm f})$. By taking advantage of the integral relationship $\int_0^{A_{sw}} p(A_{sw})dA_{sw} = P(A_{sw})$, equation (7) can be transformed into a first order homogeneous ordinary differential equation:

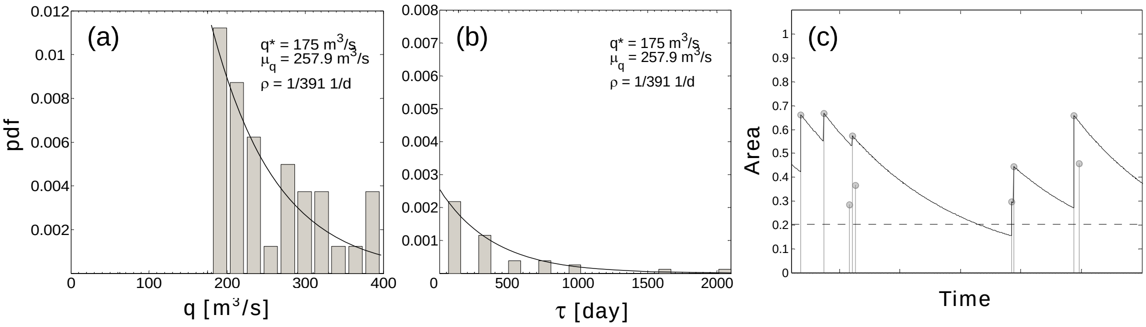

Fig. 4 Exponential distributions (a) for the event magnitudes above the threshold of $q^* = 175$ m^3/s, and (b) the related inter-arrival time between the events (b) for the process with estimated parameters. An example of a numerically generated process with event magnitude conditional dynamics is shown in (c).

$$\frac{dP^c(A_{sw})}{dA_{sw}} + \frac{(1-G^c(A_{sw}))\rho}{k_G A_{sw}} P^c(A_{sw}) = 0 \tag{8}$$

that, together with the condition for the atom of probability at $A_{sw} = A_f$ $P^c(A_{sw}) = 1 - \Theta(A_{sw} - A_f)\, p^{at}_{A_f}$ allows us to find an analytical solution of the process. This solution has a rather cumbersome expression involving a linear combination of exponential integral functions and therefore is not shown here, but can be obtained upon request from the authors.

RESULTS AND DISCUSSIONS

Before discussing the application to the Maggia case, it is instructive to show a number of example cases that can be obtained by means of the analytical solution of equation (8). These are shown in Fig. 5 as a compilation of stationary cumulative and density distribution functions corresponding to a varying colonization rate parameter k_G and inter-arrival time between the disturbance events. We obtain two limiting cases of completely deterministic asymptotic dynamics when k_G either tends to zero or becomes (hypothetically) infinite.

Case 1: $k_G \to \infty$

If the colonization rate parameter becomes very high (or infinite, at the limit) then stationarity for equation (6) is obtained by studying the limit for which:

$$\lim_{k_G \to \infty} \left\langle (A(t_i) - A_{sw})\delta(t_i)\Theta(A(t_i) - A_{sw}) \right\rangle - \left\langle k_G A_{sw} \right\rangle = 0 \tag{9}$$

This happens only for $A_{sw} \to 0$, given that the vegetation will recolonise the available surface of exposed sediment at an infinite rate. Hence, the corresponding probability distribution degenerates into an atom of finite probability of having $A_{sw} = 0$ (e.g. see Fig. 5(a.1), (b.1)). A similar situation occurs if the mean arrival rate of disturbances ρ approaches 0, which means an infinite inter-arrival time τ between events. In this case, vegetation has the time to recolonise the whole flood-plain (e.g. see Fig 5(c.1), (d.1)).

Case 2: $k_G \to 0$

If no colonization occurs, i.e. $k_G = 0$, then the stationary property for equation (6):

$$\left\langle \frac{dA_{sw}}{dt} \right\rangle = \left\langle (A(t_i) - A_{sw})\delta(t_i)\Theta(A(t_i) - A_{sw}) \right\rangle = 0 \tag{10}$$

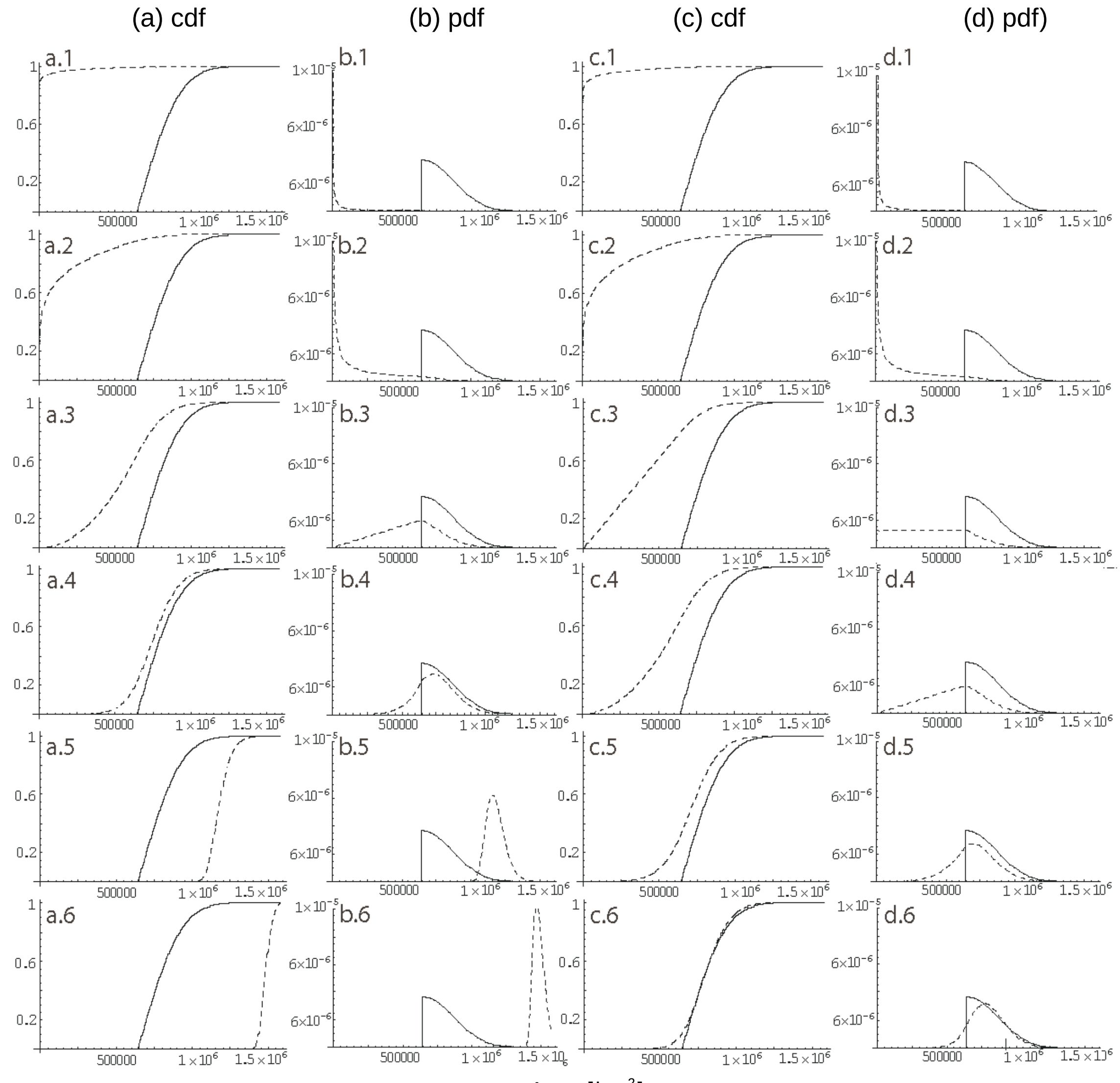

Fig. 5 Cumulative (columns a,c) and probability density (columns b,d) functions of the Master process for the variable A_{sw} for varying colonization rate parameter k_G in the range $\{0 \div \infty\}$(columns a and b) and for varying mean inter-arrival time of disturbances $\tau = 1/\rho$ (columns c and d). The continuous line is the distribution of the inundated area as flooded by streamflow events, whereas the dashed line is the distribution of the resulting process influenced by the deterministic colonization dynamics.

is automatically and forever satisfied the first time an event of magnitude $A = A_f$ does occur, which imposes $A_{sw} = A_f$. Thus, the stationary probability distribution function degenerates into an atom of finite probability of having $A_{sw} = A_f$ (e.g. see Fig 5(a.6), (b.6)).

A more likely true situation occurs for cases with intermediate values of the parameters, which lead to the other exemplary distributions shown in Fig. 5. Our simple process shows that the sediment dynamics can easily be conditioned by few fundamental parameters, either of hydrological (τ and λ) or of biological (k_G) origin. Accordingly, a fundamental result of this approach is its relevance with regard to regulated riverine environments. Specifically, our model shows that by artificially conditioning the river hydrology, a different statistical equilibrium for the flood-plain dynamics can be reached in terms of mean vegetation cover or the complementary exposed sediment surface. It is therefore interesting to show the applicative aspect of this model for the Maggia Valley case. As described in Perona *et al.* (2008) one can either estimate the

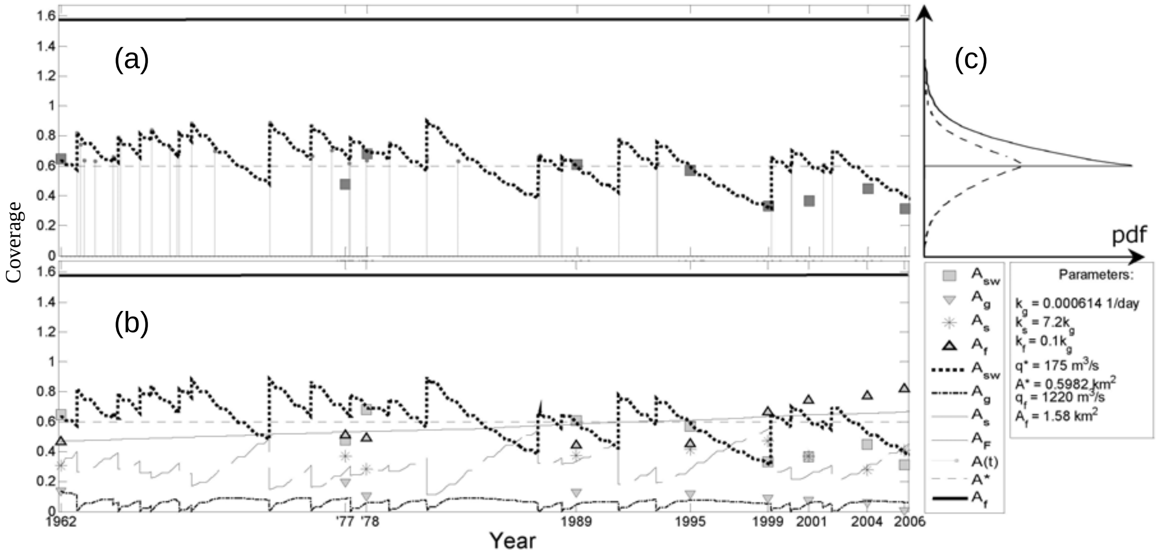

Fig. 6 Results of (a) the Master and (b) the Master–Slave models and (c) comparison with observations for the case with estimated parameters and related analytical pdf. Observations agree well for all the classes of coverage described by the models, including the slow increasing trend of the adult vegetation (forest) A_F.

model-parameters, or calibrate them from aerial photographs by using a simple error minimization technique. In this paper, we show the modelling results for the case with estimated parameters. First of all, the parameter estimation is done starting from the available aerial photographs in the post-dam period with measured streamflow, e.g. where the releases to the river during extreme events clearly assume a stochastic footprint. We determined an average colonization rate parameter k_G = 0.000614 d^{-1} by computing the exponential decay of the decreasing exposed sediment and water surface for the period 1995–1999, where no significant disturbances are observed, and fixing the threshold at q^* = 175 m^3/s. This colonization rate will be an under-estimate because it is likely that some disturbance in the period in reality did occur. We then use historical hydrological inputs to drive the model with estimated parameters, and study how it captures the main behaviour of the sediment and water dynamics (Fig 6(a)). By means of the analytical solution we can also compute the related pdf $g(A_{sw})$, the mean of which (598 000 m^2) agrees rather well with that obtained from observations (525 000 m^2). Moreover, we use the Master model to steer the Slave model and to obtain an explicit quantification of the evolution of the vegetation classes on the flood plain (Fig 6(b)). In this case, also, the results are encouraging and capture the essential features of the natural dynamics.

CONCLUSIONS

In this paper we propose a stochastic Master–Slave model for the sediment and vegetation dynamics in an Alpine regulated river, and discuss the main characteristics of the Master model. The theoretical formulation and the related analytical solution allow us to obtain the probability density function (pdf) of the exposed sediment area lumped in space. In order to validate the model we applied it to the Maggia River (southeastern Switzerland) where a historical record of aerial photographs of the studied river reach is available. Model parameters were entirely estimated from aerial photographs and from daily streamflow records, and showed good agreement with observations. We see the potential of this modelling approach in making statistical predictions of the effects of changes in the flood disturbance regime on flood plain sediment and vegetation dynamics.

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